Evolution of Gravitational Perturbations in Non-Commutative Inflation

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Abstract

We consider the non-commutative inflation model of [3] in which it is the unconventional dispersion relation for regular radiation which drives the accelerated expansion of space. In this model, we study the evolution of linear cosmological perturbations through the transition between the phase of accelerated expansion and the regular radiation-dominated phase of Standard Cosmology, the transition which is analogous to the reheating period in scalar field-driven models of inflation. If matter consists of only a single non-commutative radiation fluid, then the curvature perturbations are constant on super-Hubble scales. On the other hand, if we include additional matter fields which oscillate during the transition period, e.g. scalar moduli fields, then there can be parametric amplification of the amplitude of the curvature perturbations. We demonstrate this explicitly by numerically solving the full system of perturbation equations in the case where matter consists of both the non-commutative radiation field and a light scalar field which undergoes oscillations. Our model is an example where the parametric resonance of the curvature fluctuations is driven by the oscillations not of the inflaton field, but of the entropy mode.

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I. INTRODUCTION

Non-commutativity of space and time is a general prediction for an effective field theory coming from string theory [1]. This leads to a maximum wavenumber p_c for any field which is subject to this uncertainty. In turn, this leads to a dispersion relation which is modified at high densities. In particular, regular radiation is described by such a modified dispersion relation [2]. As shown recently in [3], the dispersion relation will likely contain two branches (two frequencies for any wavenumber smaller than the maximal momentum). This two-branch structure may be related to the T-duality symmetry of string theory which has been made use of in a different context in string cosmology in [4]. At high temperatures, both branches of the dispersion relation are occupied, whereas at low temperatures only the lower branch, the branch which for $p \ll p_c$ has the regular linear form, is occupied.

In [3] it was realized that for suitable classes of dispersion relations, regular radiation leads to inflationary expansion of space at high temperatures. The reason is that as space expands and the physical momentum of a fixed comoving mode decreases, the energy of the mode increases. This increasing energy leads (in the context of a background described by Einstein gravity) to accelerated expansion. We called this model "non-commutative inflation".

Recently [5] we have studied the generation of cosmological fluctuations in non-commutative inflation. In contrast to simple scalar field-driven inflationary models, the fluctuations are thermal of origin and not quantum vacuum fluctuations, the reason being that matter in our scenario is a thermal bath rather than an ultracold Bose condensate of a scalar field. In the limit that the accelerated expansion is almost exponential, the spectrum of fluctuations is almost scale-invariant, as should be expected from the general symmetry arguments of [6].

An interesting aspect of non-commutative inflation is that the transition between the period of accelerated expansion and the radiation phase of standard cosmology is straightforward: matter always is in regular radiation. As the universe expands, the equation of state of this radiation changes from being that of an accelerated universe early on to that of ordinary radiation. Thus, there is no need for any period of "reheating", a rather non-trivial phase in usual scalar field-driven inflationary models.

In this paper we study the evolution of the fluctuations through the transition between

the inflationary phase and the phase when the equation of state changes to that of ordinary radiation. Since there are no oscillations in the equation of state of that matter giving rise to inflation (in scalar field-driven inflation the phase of accelerated expansion ends with oscillations of the scalar field giving rise to inflation which translates into oscillations in the equation of state of matter). As a consequence, there is no parametric resonance instability of matter [7, 8] or metric [9, 10, 11, 12] fluctuations in this phase, at least in the absence of other matter fields which undergo oscillations.

In models of matter beyond the Standard Model, in particular in models motivated by superstring theory, there are many scalar matter fields which are expected to undergo oscillations at the end or after the period of inflation. These fields lead to entropy modes. Here, we will study the evolution of cosmological fluctuations in the presence of such oscillating entropy modes (oscillating "moduli" fields). We find that these entropy modes can indeed lead to parametric resonance of the curvature fluctuations on super-Hubble scales.

The importance of parametric resonance instabilities in the process of reheating at the end of inflation was first recognized in [7] (see also [8]). The resonance of matter fluctuations during reheating in scalar field-driven inflation was then studied in more detail in [13] (where the name "preheating" for this resonance phenomenon was coined) and [14]. A detailed discussion of the efficient broad-band and the much less efficient narrow-band resonances was given in [15]. It was first conjectured in [9] that super-Hubble (but sub-horizon) scales metric fluctuations may also be parametrically amplified. In particular, there are no causality constraints [10] which prohibit this (recall that in inflationary cosmology the horizon is larger than the Hubble radius by a factor of $\exp(N_e)$, where N_e is the number of e-foldings of the inflationary phase). However, as shown in [10], in the case of purely adiabatic fluctuations there is no resonance at linear order in perturbation theory. However, it was then established in [11, 12] that oscillations in the equation of state of matter can lead to an increase of the curvature fluctuations on super-Hubble scales in the presence of low mass (mass smaller than the Hubble rate during inflation) entropy modes.

In previous work, most of the attention has focused on oscillations of the inflaton driving the parametric resonance of the entropy modes of cosmological fluctuations, which in turn feeds the exponential growth of the curvature perturbation via the usual sourcing of curvature fluctuations via an entropy inhomogeneity. In the context of non-commutative inflation, it is the background value of the entropy field which undergoes oscillations. Here,

we show that such oscillations may also be able to excite a parametric resonance for entropy fluctuations.

The outline of this article is as follows: In the following section we give a brief review of non-commutative inflation. Then, in Section 3, we study the growth of cosmological fluctuations during the transition between accelerated expansion and the usual radiation phase in the case of a single component of matter, the radiation. As expected, we find constant curvature fluctuation on super-Hubble scales. The key section of our paper is Section 4, in which we study the equations of motion for cosmological fluctuations for non-commutative radiation coupled to an oscillating scalar field (the modulus field acting as the entropy mode). We derive the relevant perturbation equations, solve them numerically, and give some analytic insight into the solutions. We conclude with a discussion of our results.

II. NONCOMMUTATIVE INFLATION

The starting point of the non-commutative inflation model of [3] is the modified dispersion relation for massless particles

$$E^2 - p^2c^2 f(E)^2 \,=\, 0 \tag{1}$$

which results from the non-commutativity of space and time. Here,

$$f(E) \,=\, 1 + (\lambda E)^\alpha \,. \tag{2}$$

Note that p and E denote momentum and energy, respectively, $\alpha \geq 1$ is a positive constant, and the length scale λ determines the maximal momentum p_c. For $\alpha > 1$ the dispersion relation has two branches. On the upper branch, the energy increases as the momentum decreases while the universe is expanding. It is this behavior which, for a range of values of α explored in [3], leads to inflationary expansion of space when the usual Friedmann-Robertson-Walker equations for the coupling of the background space-time to matter are used.

The modification of the dispersion relation leads to a deformed thermal spectrum [2]. The energy density ρ is given by

$$\rho \,=\, \frac{1}{\pi^2}\int \frac{E^3}{e^{E/T}-1}\frac{1}{f^3}\left|1-\frac{f'E}{f}\right| \,, \tag{3}$$

and the expression for the pressure $\mathcal{P}$ is

$$\mathcal{P} = \frac{1}{3}\int \frac{\rho(E)dE}{1-\frac{f'E}{f}}, \tag{4}$$

In the high energy limit, $\lambda E \gg 1$, $f'E/f \simeq \alpha$ and $\rho \propto T$. This approximation leads to a nearly constant equation of state parameter $w \equiv \mathcal{P}/\rho$.

Regular commutative inflation has no intrinsic entropy fluctuations since

$$c_a^2 \equiv \frac{\dot{\mathcal{P}}}{\dot{\rho}} = c_s^2 \equiv \frac{\delta\mathcal{P}}{\delta\rho} \tag{5}$$

where δ indicates spatial variations. The same result holds for non-commutative radiation. This can be seen by taking on one hand the time derivatives of Eqs. (3) and (4), and on the other hand the infinitesimal spatial variations of these quantities. The vanishing of the intrinsic entropy fluctuations will be important later on.

If the background geometry is described by the Einstein action, the Friedmann and energy conservation equations take their usual form

$$\left(\frac{\dot{a}}{a}\right)^2 \equiv H^2 = \frac{8\pi G}{3}\rho, \tag{6}$$

$$\dot{\rho} = -3H(1+w)\rho\,. \tag{7}$$

Inserting the expressions (3) and (4) into the above equations, it follows that at high temperatures, a period of accelerated expansion results provided that the parameter α is suitably chosen [3] (values of α slightly larger than 1). The resulting evolution of the equation of state parameter w and of the adiabatic sound speed c_a^2 is shown in Fig. 1.

Inflation has to last more than 60 number of e-foldings (this number assumes that the energy scale at which inflation takes place is close to its upper bound) to solve the problems of Standard Cosmology which inflation was designed to address. If noncommutative inflation ends roughly when $\lambda T_{end} \simeq 1$, then the number N_e of e-foldings is given by

$$N_e = \ln[a(t_{end})/a(t_i)] = \int_{t_i}^{t_{end}} H dt = \int_{\lambda T_{end}}^{\lambda T_i} \frac{d\rho/dT}{3(1+w)\rho} dT\,. \tag{8}$$

In the high energy limit, we obtain

$$N_e \simeq \frac{1}{3(1+w)} \ln\frac{\lambda T_i}{\lambda T_{end}} = \frac{\ln \lambda T_i}{3(1+w)}, \tag{9}$$

where we have used $\lambda T_{end} \sim 1$.

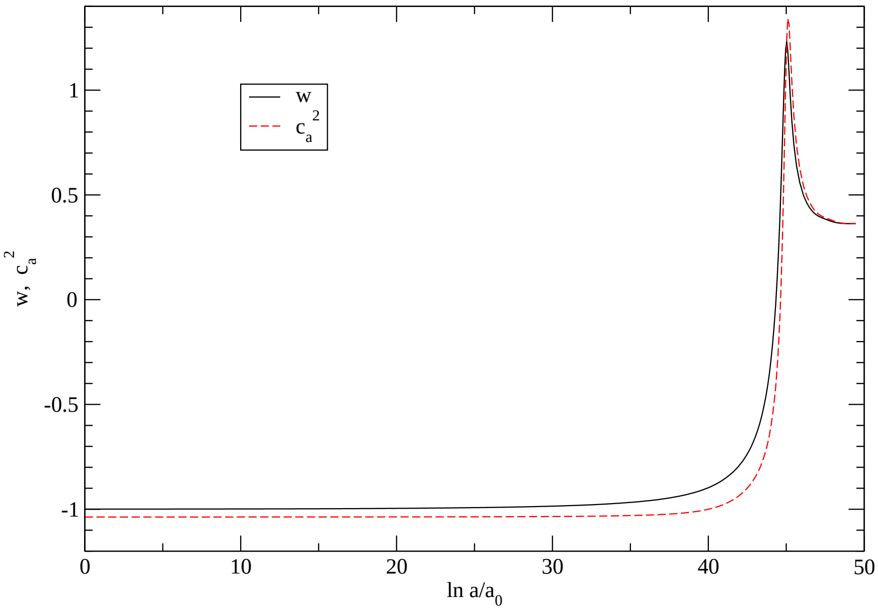

FIG. 1: Evolution of the equation of state parameter w and of the adiabatic sound speed c_a^2 for a noncommutative fluid during and after the period of inflation. The value of α is set to 1.099.

III. EVOLUTION OF NONCOMMUTATIVE FLUID PERTURBATIONS

A. Equations of motion

In this section we will take matter to consist only of the non-commutative fluid. In this case, since there are no entropy fluctuations we expect the curvature fluctuations to be conserved on super-Hubble scales. In addition, since there is no oscillating matter field, there cannot be any parametric resonance effects.

We will work in longitudinal gauge to study the linearized equations of motion for cosmological perturbations (see [16, 17] for comprehensive reviews of the theory of cosmological perturbations and [18] for a pedagogical overview). In this gauge, the metric takes the form

$$ds^2 = -(1+2\Phi)dt^2 + a^2(1-2\Psi)\gamma_{ij}dx^i dx^j \,, \tag{10}$$

where γ_{ij} is the spatial part of the background metric which we in the following will take to be Euclidean. The perturbations are described by Φ (the Bardeen [19] potential) and Ψ, both functions of space and time.

If matter consists only of the noncommutative (NC) fluid, the perturbed energy-

momentum tensor can be written as

$$\delta T^0{}_0 = -\delta\rho, \quad \delta T^0{}_i = \delta q_i, \quad \delta T^i{}_j = \delta\mathcal{P}\delta^i_j, \tag{11}$$

where we have assumed vanishing anisotropic stress.

The perturbed Einstein equations are

$$3H(H\Phi + \dot{\Psi}) - \frac{1}{a^2}\nabla^2\Psi \;=\; -4\pi G\delta\rho, \tag{12}$$

$$H\Phi + \dot{\Psi} \;=\; -4\pi G\delta q, \tag{13}$$

$$\ddot{\Psi} + 3H\dot{\Psi} + H\dot{\Phi} + (2\dot{H} + 3H^2)\Phi - \frac{1}{3a^2}\nabla^2(\Psi - \Phi) \;=\; 4\pi G\delta\mathcal{P}. \tag{14}$$

The vanishing of the anisotropic stress tensor makes it possible to set $\Phi = \Psi$.

The conservation of energy-momentum tensor for the NC fluid leads to the following equations for $\delta\rho$ and δq,

$$\delta\dot{\rho} + 3H(\delta\rho + \delta\mathcal{P}) - 3\rho(1+w)\dot{\Phi} + \frac{1}{a^2}\nabla^2\delta q \;=\; 0, \tag{15}$$

$$\delta\dot{q} + 3H\delta q + \rho(1+w)\Phi + \delta p \;=\; 0. \tag{16}$$

Combining Eqs. (12) and (14) and working in momentum space $\nabla^2\Phi \to -k^2\Phi_k$, we can obtain a single differential equation

$$\ddot{\Phi}_k + (4 + 3c_s^2)H\dot{\Phi}_k + \left(\frac{k^2c_s^2}{a^2} + 2\dot{H} + 3H^2(1 + c_s^2)\right)\Phi_k \;=\; 0\,, \tag{17}$$

where we have used $\delta\mathcal{P} = c_s^2\delta\rho$.

For adiabatic fluctuations, and in particular in our single perfect fluid system, $c_s^2 = c_a^2$. However, in non-adiabatic fluids, for example systems with several fluids or scalar fields, there may exist intrinsic isocurvature perturbations which are denoted by

$$p\Gamma \;=\; (c_s^2 - c_a^2)\delta\rho. \tag{18}$$

The Sasaki-Mukhanov variable [20, 21] for a fluid in terms of which the action for cosmological perturbations has canonical form, and which is thus useful for the quantization of these fluctuations, can be written as $v = ac_s^{-1}Q_f$ where [17, 22],

$$Q_f \;=\; \frac{1}{\sqrt{\rho + \mathcal{P}}}\left(\delta q - \frac{\rho + \mathcal{P}}{H}\Phi\right). \tag{19}$$

If matter is a scalar field, this gauge-invariant quantity becomes

$$Q_\phi \;=\; \delta\phi - \frac{\dot{\phi}}{H}\Phi. \tag{20}$$

In usual scalar field-driven inflationary models, the scalar field begins to oscillate about the minimum of its potential after the end of inflation. These oscillations lead to a singularity if Eq. (17) is used because of the $1/\dot{\phi}$ terms in the coefficients. To avoid this singularity during the preheating phase, the Sasaki-Mukhanov variables must be used instead of Φ [23]. Even in the absence of oscillations, if the change in the equation of state of the background in smooth, it is safer to use the Sasaki-Mukhanov variables instead of Φ (see e.g. [24, 25]).

The situation, however, is different in our fluid-driven inflationary model. The speed of sounds c_s^2 is negative during the inflationary period. Then, after the end of inflation, c_s^2 crosses zero and evolves to its final positive value $c_s^2 = 1/3$ (see Fig. 1). The evolution equation for Q_f, Eq. (A2), contains $(c_s^2)^{-1}$ terms which diverge when $c_s^2 = 0$. Therefore it is convenient to use the Φ equation of motion (17) to evolve the fluctuations in our fluid-driven inflation model in order to avoid the singularity at $c_s^2 = 0$.

The curvature perturbation ζ on uniform energy density hyper-surfaces is related to the Sasaki-Mukhanov variable as

$$\zeta = \Phi - \frac{H}{\dot{H}}(\dot{\Phi} + H\Phi) = -\frac{H}{\sqrt{\rho + \mathcal{P}}} Q_f. \tag{21}$$

The evolution equation for ζ_k is

$$\dot{\zeta}_k = \frac{H}{\dot{H}} \frac{k^2 c_s^2}{a^2} \Phi_k - \frac{H}{\rho(1+w)} p\Gamma. \tag{22}$$

In the case of adiabatic fluids when ($p\Gamma = 0$), then for long-wavelength perturbations ζ_k is conserved.

B. Numerical results

We performed numerical calculations of the evolution of the linear cosmological perturbations during and after NC inflation. The simulations were done using the value $k = 5a_i H_i$ for the comoving momentum (the subscripts i standing for the initial time of the analysis) and started from $\lambda T_i = 9900$ to satisfy the condition of Eq. (9). Thus, the modes are inside the Hubble radius in the initial stage of the inflationary period. Quantum vacuum initial conditions for the fluctuations were used [30]. To implement these initial conditions, we postulate the usual harmonic oscillator vacuum initial conditions for the Fourier modes of the variable v in terms of which the action for fluctuations has canonical kinetic term. This

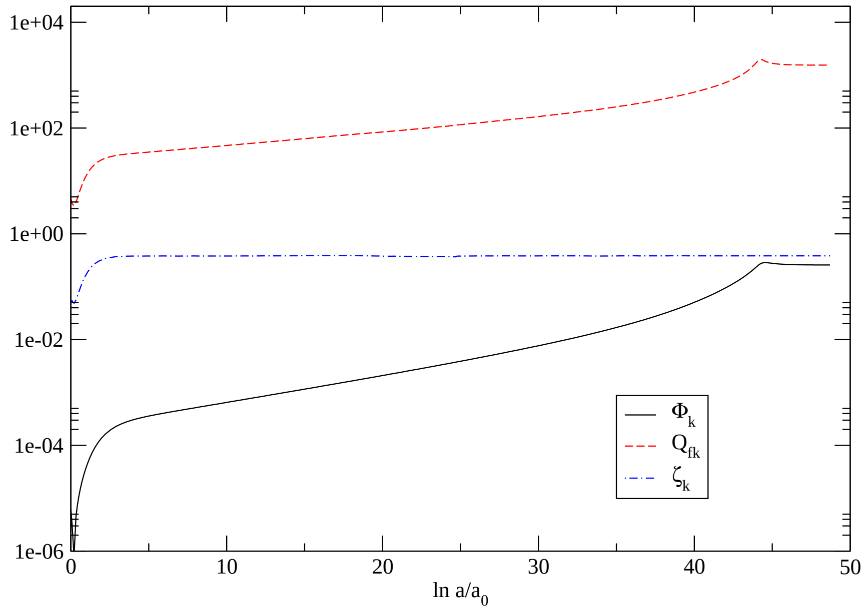

FIG. 2: Evolution of the Sasaki-Mukhanov variable Q_{fk} in the case when matter is a single NC fluid. Also shown are the curvature perturbation ζ_k and the Bardeen potential Φ_k. The simulations starts during the inflationary period and the results shown are for the values $\lambda^{-1} = 10^{-4} m_{pl}$ and $\alpha = 1.099$.

variable is given by

$$v_k \;=\; \frac{a}{c_s} Q_{fk} \;=\; -z\zeta_k, \tag{23}$$

where the variable z is

$$z \;=\; \frac{a\sqrt{\rho + \mathcal{P}}}{H c_s}\,. \tag{24}$$

The variable v_k obeys the following wave equation

$$v_k'' + \left(k^2 c_s^2 - \frac{z''}{z}\right) v_k \;=\; 0, \tag{25}$$

where a prime denotes the derivative with respect to conformal time η ($dt = a d\eta$). The initial conditions for the k'th Fourier mode v_k of v are given by

$$v_k(t_i) \;=\; \frac{1}{\sqrt{2kc_s}}, \quad \dot{v_k}(t_i) \;=\; -i\sqrt{\frac{kc_s}{2}}. \tag{26}$$

Then, from Eqs. (21) and (22), we obtain the initial conditions for Φ (suppressing the index k for now)

$$\Phi_i = -\frac{\dot{H}}{H}\frac{a^2}{k^2 c_s^2}\left(\frac{v_i}{z}\right)^{\cdot} = 4\pi G\frac{Hz^2}{k^2}\left(\frac{v_i}{z}\right)^{\cdot}, \tag{27}$$

$$\frac{H}{a}\left(\frac{a}{H}\Phi_i\right)^{\cdot} = \frac{\dot{H}}{H}\frac{v_i}{z} = -4\pi G\frac{\rho(1+w)}{Hz}v, \tag{28}$$

where $v_i = v(t_i)$.

We set the cutoff mass $\lambda^{-1} = 10^{-4}m_{pl}$ to be consistent with the observational data [3, 5] and use $\alpha = 1.099$, a value which gives an inflationary background. With the initial conditions described above and these parameter values, we numerically solve the Eq. (17).

As shown in Section 2, the speed of sound is given by $c_s^2 = c_a^2$. Thus, during the inflationary phase c_s^2 is negative. This leads to an instability of short wavelength fluctuations. Since the focus of this paper is on the evolution of the fluctuations on super-Hubble scales, and since - according to the prescription of [3, 5] - we should impose initial conditions for the fluctuations at the typical thermal wavelength, and modes thus do not spend a long time on sub-Hubble scales, we will not worry about this instability in this paper.

In Fig. 2, we display the evolution of Φ_k, Q_{fk} and ζ_k during and after inflation. Fig. 2 shows that the curvature perturbation ζ_k stays constant for super-Hubble scales both during inflation and after the end of inflation. This is the same behavior as holds for adiabatic fluctuations in scalar field-driven inflationary models. Meanwhile, the time-dependent equation of state causes the Bardeen potential Φ_k and also Sasaki-Mukhanov variable Q_{fk} to increase for super-Hubble scales during inflation as time increases. We wish to emphasize that, as expected, there is no amplification of the amplitude of curvature perturbations at the end of inflation. This can be understood from the fact that in Eq. (25), the function z''/z term does not include any oscillating terms at the end of inflation.

Our numerical analysis (Fig. 2) also shows that the curvature perturbation ζ_k as well as Φ_k and Q_{fk} increase on sub-Hubble scales . This is unlike what happens in the usual scalar field-driven inflation model. The difference, as discussed above, is due to the fact that in our case the sound speed c_s^2 is negative during inflation.

IV. EVOLUTION OF NONCOMMUTATIVE FLUID PERTURBATIONS INCLUDING AN OSCILLATING SCALAR FIELD

Particle physics models beyond the Standard Model, in particular those based on supersymmetry, supergravity and superstring theory, typically contain many scalar fields. These fields will be displaced from the minima of their potentials by quantum fluctuations during the period of inflation. After inflation, they will start to oscillate about their minima. In this section we will study the effects of these oscillations on the parametric excitation of entropy fluctuations on super-Hubble scales.

In different contexts, cosmological perturbations in models with both scalar fields and fluids have been studied before. For example, in warm inflation models [26], the scalar inflaton field is coupled to a thermal fluid. Since the scalar field evolution is typically overdamped, there will be no parametric resonance effects. The growth of fluctuations in this context has been considered e.g. in [27]. Our situation is different in that the dynamics is driven by the fluid rather than by the scalar field. In a more general context, perturbations in a system consisting of a scalar field plus an ideal gas were recently also considered in [28], in particular with applications to quintessence cosmology in mind.

In the following, we first discuss the equations of motion for the cosmological fluctuations, and then present our numerical results.

A. Equations of motion

The background scalar field ϕ satisfies the Klein-Gordon equation

$$\ddot{\phi} + 3H\dot{\phi} + V_\phi \; = \; 0, \tag{29}$$

where $V_\phi = \partial V/\partial\phi$. In the presence of both a fluid and a scalar field, the background Friedmann equation is

$$H^2 \; = \; \frac{8\pi G}{3}\left(\rho_f + \frac{1}{2}\dot{\phi}^2 + V(\phi)\right). \tag{30}$$

where subscript f denotes a fluid quantity.

In Fig. 3, we plot the evolution of a free scalar field ϕ in a universe dominated by non-commutative radiation. Different plots are for different values of the scalar field mass m.

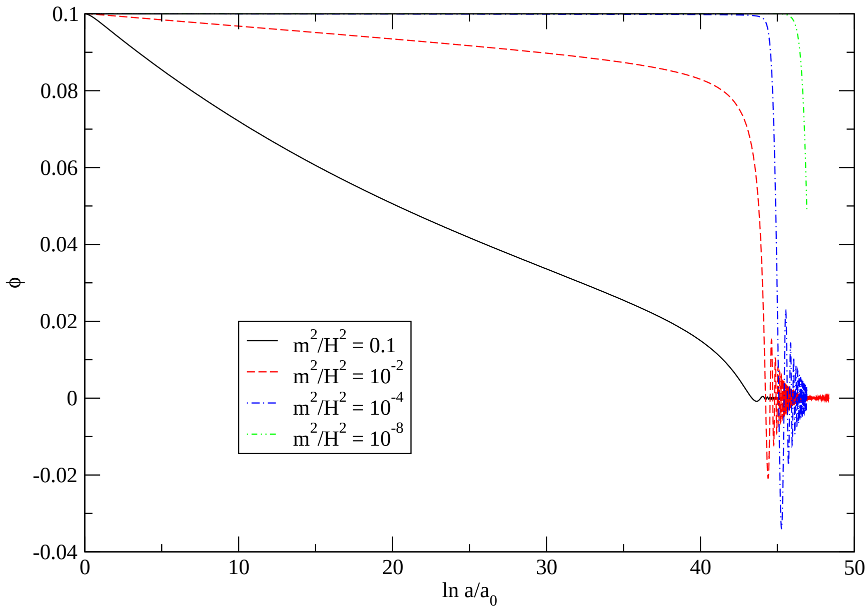

FIG. 3: Evolution of the scalar field during and after the period of NC inflation for various values of the scalar field mass m. The parameters $\lambda^{-1} = 10^{-4} m_{pl}$ and $\alpha = 1.099$ were used.

The total perturbed energy density, pressure and momentum are

$$\delta\rho = \delta\rho_f + \delta\rho_\phi, \quad \delta\mathcal{P} = \delta\mathcal{P}_f + \delta\mathcal{P}_f,$$
$$\delta q = \delta q_f - \dot{\phi}\delta\phi, \tag{31}$$

where

$$\delta\rho_\phi = \dot{\phi}\delta\dot{\phi} - \dot{\phi}^2\Phi + V_\phi\delta\phi, \tag{32}$$
$$\delta\mathcal{P}_\phi = \dot{\phi}\delta\dot{\phi} - \dot{\phi}^2\Phi - V_\phi\delta\phi. \tag{33}$$

By inserting the metric including cosmological fluctuations into the curved space-time Klein-Gordon equations, one obtains the following equation for the fluctuation of the scalar field

$$\delta\ddot{\phi}_k + 3H\delta\dot{\phi}_k + \left(\frac{k^2}{a^2} + V_{\phi\phi}\right)\delta\phi_k = 4\dot{\Phi}_k\dot{\phi} - 2V_\phi\Phi_k\,. \tag{34}$$

By subtracting c_s^2 times the $0-0$ perturbed Einstein equation from the space-space perturbed Einstein equation, one obtains the following evolution equation for the Bardeen potential

$$\ddot{\Phi}_k + (4+3c_s^2)H\dot{\Phi}_k + \left[\frac{k^2c_s^2}{a^2} + 2\dot{H} + 3H^2(1+c_s^2) + 4\pi G(1-c_s^2)\dot{\phi}^2\right]\Phi_k$$
$$= 4\pi G[(1-c_s^2)\dot{\phi}\delta\dot{\phi}_k - (1+c_s^2)V_\phi\delta\phi_k], \tag{35}$$

where

$$\dot{H} = -4\pi G(\rho_f(1+w_f) + \dot{\phi}^2). \tag{36}$$

Note that in the above c_s^2 is the speed of sound of the fluid component.

The isocurvature perturbation for a multi-component system (the components being labelled by indices $i, j, ...$) takes the form [16]

$$p\Gamma = \sum_i (c_{si}^2 - c_{ai}^2)\delta\rho_i + \frac{1}{2}\sum_{i,j}\frac{h_i h_j}{h}(c_{ai}^2 - c_{aj}^2)S_{ij}, \tag{37}$$

where we have used the abbreviations

$$h = \rho + \mathcal{P}, \quad h_i = \rho_i + \mathcal{P}_i\,. \tag{38}$$

The first term in Eq. (37) represents the non-adiabatic pressures of the individual components and the second term the relative isocurvature perturbations between the different components. If energy transfer between the components is neglected, S_{ij} becomes

$$S_{ij} = \frac{\delta\rho_i}{\rho_i + \mathcal{P}_i} - \frac{\delta\rho_j}{\rho_j + \mathcal{P}_j}. \tag{39}$$

In our case we consider one fluid and one scalar field, and the relevant term, the expression for $S_{f\phi}$ is

$$\begin{aligned} S_{f\phi} &= -\frac{3H^2\Phi_k + 3H\dot{\Phi}_k + \frac{k^2}{a^2}\Phi_k}{4\pi G\rho_f(1+w_f)} - \frac{\rho_f(1+w_f) + \dot{\phi}^2}{\rho_f(1+w_f)\dot{\phi}^2}\delta\rho_\phi \\ &= -\frac{1}{c_s^2\sqrt{\rho_f(1+w)}}\left(\dot{Q}_f + \frac{3}{2}H(1-c_a^2)Q_f\right) - \frac{1}{\dot{\phi}^2}(\dot{\phi}\dot{Q}_\phi + V_\phi Q_\phi) \\ &\quad -\frac{4\pi G}{H}\frac{1-c_s^2}{c_s^2}(\dot{\phi}Q_\phi - \sqrt{\rho_f(1+w)}Q_f)\,. \end{aligned} \tag{40}$$

Then, from Eq. (37), the relative isocurvature perturbation is

$$p\Gamma_{rel} = \frac{1}{2}\frac{\rho_f(1+w_f)\dot{\phi}^2}{\rho_f(1+w_f) + \dot{\phi}^2}(c_{af}^2 - c_{a\phi}^2)S_{f\phi}, \tag{41}$$

where the speed of sound of the scalar field component is

$$c_{a\phi}^2 = \frac{\dot{p}_\phi}{\dot{\rho}_\phi} = 1 + \frac{2V_\phi}{3H\dot{\phi}}. \tag{42}$$

As we have discussed in Section 2, there is no non-adiabatic pressure for the NC fluid. The non-adiabatic pressure for the scalar field can be written as

$$
\begin{aligned}
p\Gamma_{int} &= -\frac{2V_\phi}{3H\dot{\phi}}\delta\rho_\phi \\
&= -\frac{2V_\phi}{3H\dot{\phi}}(\dot{\phi}\dot{Q}_\phi + V_\phi Q_\phi) + \frac{V_\phi\dot{\phi}}{\rho}(\dot{\phi}Q_\phi - \sqrt{\rho_f(1+w_f)}Q_f).
\end{aligned} \tag{43}
$$

The sum of Eqs. (41) and (43) gives the total non-adiabatic pressure which is the source term in the equation of motion for the curvature perturbation ζ (see Eq. (22)).

In the case of a multi-component system, the variable ζ which geometrically represents the curvature perturbation on the uniform energy density slices is given by [29]

$$
\begin{aligned}
\zeta &= \frac{\sum_i \dot{\rho}_i\zeta_i}{\sum_i \dot{\rho}_i} \\
&= H\frac{-\sqrt{\rho_f(1+w_f)}Q_f + \dot{\phi}Q_\phi}{\rho_f(1+w_f)+\dot{\phi}^2},
\end{aligned} \tag{44}
$$

and its equation of motion is given by the generalization of (22) as

$$
\dot{\zeta}_k = \frac{H}{\dot{H}}\frac{k^2c_s^2}{a^2}\Phi_k - \frac{H}{\rho_f(1+w_f)+\dot{\phi}^2}p\Gamma \tag{45}
$$

From this, we see that in the case of the NC fluid coupled to a scalar field, there is a non-vanishing source for ζ_k which is not suppressed on large scales (the second term on the right hand side of Eq. (45)), and if the scalar field is oscillating, there is a possibility of parametric resonance enhancement of ζ_k. We will now turn to a numerical study of this question.

B. Numerical Calculations

We have evolved Eqs. (34) and (35) numerically for a massive scalar field potential

$$
V(\phi) = \frac{1}{2}m^2\phi^2. \tag{46}
$$

The simulations began when the perturbation modes are still inside the horizon, and the scalar field fluctuations were started in their vacuum state given by

$$
\delta\phi_k(t_i) = \frac{1}{\sqrt{2k}}, \quad \delta\dot{\phi}_k(t_i) = -i\sqrt{\frac{k}{2}}. \tag{47}
$$

Since we are considering initial conditions for which the energy density is dominated by the NC fluid, i.e. $\rho_f \gg \rho_\phi$ during the early stages of the inflation, we used the same initial conditions for Φ as in Eq. (28).

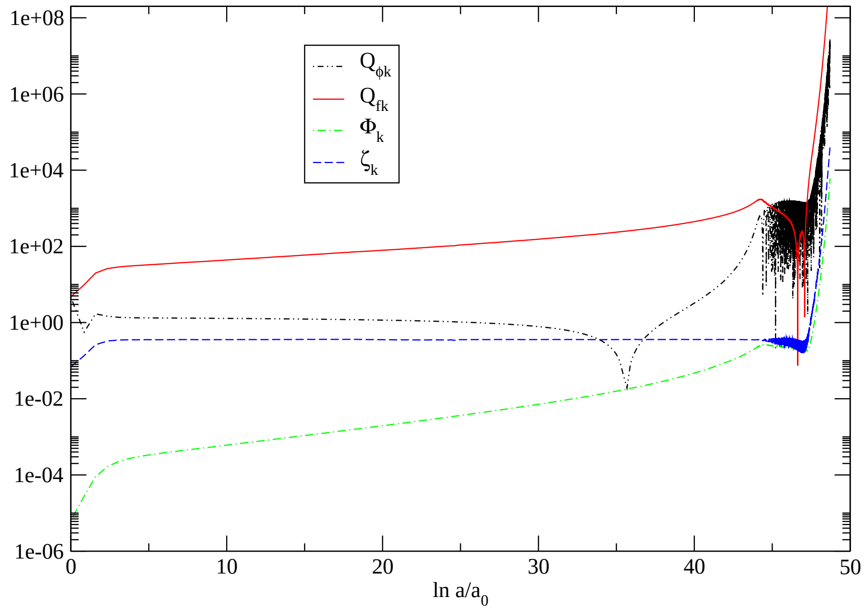

FIG. 4: Evolution of the individual Q variables, namely Q_{fk} for the NC fluid and $Q_{\phi k}$ for the scalar field, the Bardeen potential Φ_k, and curvature perturbation ζ_k, in our model of NC inflation. The results are for the choice of the NC inflation parameters $\lambda^{-1} = 10^{-4} m_{pl}$ and $\alpha = 1.099$. In the scalar field sector, we set the initial value of ϕ as $\phi_i = 0.1 m_{pl}$, $\dot{\phi}_i = 0$, and $m^2/H^2 = 10^{-2}$.

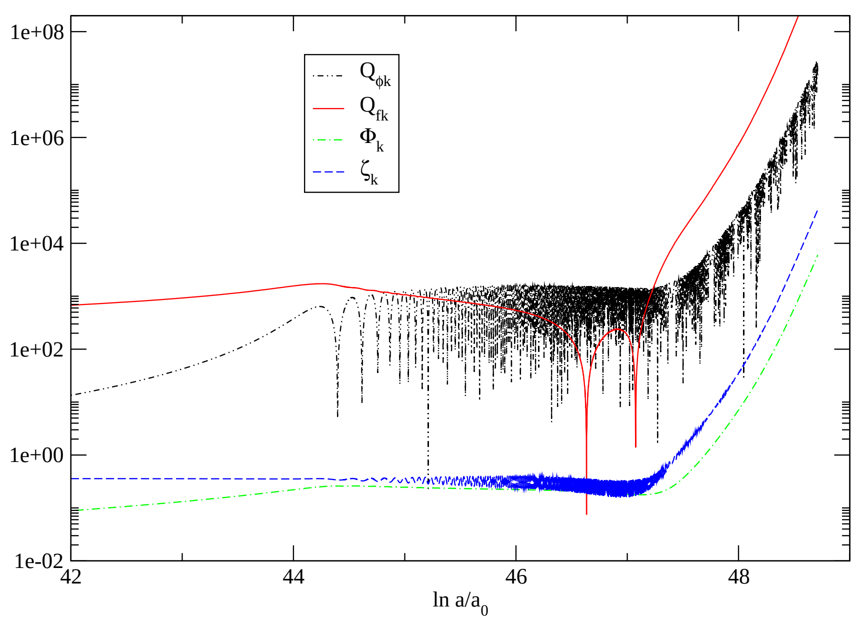

FIG. 5: A closer look at the region of oscillations in the simulation of Fig. 4.

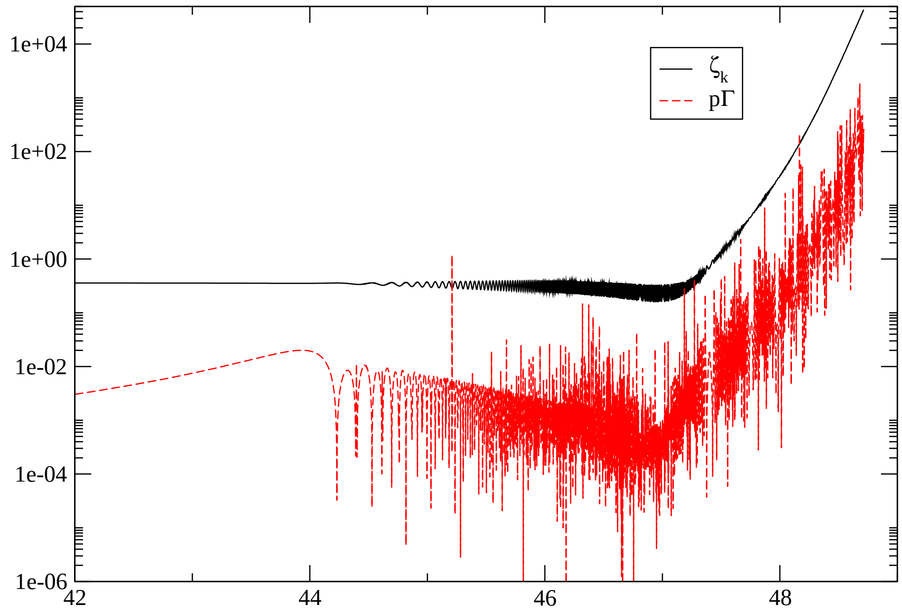

FIG. 6: Comparison of the curvature perturbation ζ_k and the isocurvature fluctuation $p\Gamma$ in the simulation of Fig. 4.

In Fig. 3 we plot the evolution of the background scalar field during and after inflation for different values of the mass parameter of the scalar field. We used the NC fluid parameters $\lambda^{-1} = 10^{-4}m_{pl}$ and $\alpha = 1.099$, and chose the initial value of the background scalar field as $\phi_i = 10^{-1}m_{pl}$ with vanishing initial velocity. We considered different values of the mass.

For $m > H$, the scalar field oscillations are rapidly damped during inflation, and no oscillations arise during the post-inflationary era. For $m \ll H$, the scalar field is over-damped during and immediately after inflation and thus hardly rolls at all during the time interval of our simulations. The interesting case for our purpose arises when the mass m is slightly but not much smaller than H. In this case, the scalar field rolls slowly during inflation and, when the inflationary phase ends and the Hubble parameter starts to decrease, the slowly decreasing Hubble parameter becomes comparable in value to the mass of the scalar field and then begins to oscillate about the minimum of its potential. The time scale of the oscillations becomes smaller than the Hubble time as H decreases. This provides the conditions where parametric resonance excitation of the entropy modes is expected [12].

The evolution of Q_f, Q_ϕ, Φ_k and ζ_k are plotted in Fig. 4 for $m^2 = 10^{-2}H^2$ and $\phi_i = 10^{-1}m_{pl}$ and for the same parameter values of λ andα as in Fig. 3. With these parameter values, parametric amplification of the curvature perturbation results after the end of the

inflationary phase.

To see the region in which parametric resonance occurs in detail, in Fig. 5 we focus on the time interval where the scalar field oscillations occur. For a value of the mass given by $m^2 = 10^{-2}H^2$, the oscillation of the background scalar field during this time interval are seen (see Fig. 3) to cause oscillations of the scalar field fluctuation variable Q_ϕ. Via the gravitational coupling of the two matter components, this leads to the parametric amplification of the fluid fluctuation variable Q_f.

To understand the parametric resonance after inflation in our NC fluid model of inflation with an oscillating scalar matter field, we must consider the coupled system of evolution equations for Q_f and Q_ϕ which are given in the Appendix (Eqs. (A1) and (A2)). We assume that the time scale of the resonance is short compared to the Hubble time H^{-1}. In this case, we can neglect that Hubble damping terms in the evolution equations. We will also drop the other terms involving only first derivatives of the Q variables - we later check the self-consistency of this approximation - and we will also drop coefficients of Q_f and Q_ϕ terms which are obviously suppressed compared to other coefficients. Then, we obtain approximate equations

$$\ddot{Q}_{fk} + \left[\frac{k^2c_s^2}{a^2} + 6\pi GA\rho_f - 6\pi GB\dot{\phi}^2\right]Q_{fk} - 4\pi G\sqrt{\rho_f(1+w_f)}[C + 3\dot{\phi}D]Q_{\phi k} \simeq 0\,, \quad (48)$$

where

$$A \equiv (1+w)(1+3c_s^2) + (1-c_s^2)(1+c_s^2) - 2(1+w_f)^2\frac{\rho_f}{\rho}, \quad (49)$$

$$B \equiv -c_s^2(1-c_s^2) + (1+w)(1+c_s^2)\frac{\rho_f}{\rho}, \quad (50)$$

$$C \equiv (1+c_s^2)\frac{V_\phi}{H}, \quad (51)$$

$$D \equiv (1+c_s^2) - (1+w)\frac{\rho_f}{\rho}\,, \quad (52)$$

and

$$\ddot{Q}_{\phi k} + \left[\frac{k^2}{a^2} + V_{\phi\phi} + 16\pi GV_\phi\frac{\dot{\phi}}{H}\right]Q_{\phi k} + 8\pi G\sqrt{\rho_f(1+w_f)}\left[\frac{V_\phi}{H} + \epsilon\dot{\phi}\right]Q_{fk} \simeq 0\,, \quad (53)$$

where

$$\epsilon = 3 + \frac{3}{4}\frac{(1-c_s^2)^2}{c_s^2} - \frac{3}{4}\frac{(1+c_s^2)(1+w_f)}{c_s^2}\frac{\rho_f}{\rho}. \quad (54)$$

Note that at the end of the inflationary phase, $\rho_f/\rho \sim 1/2$ because of $\rho_f \simeq \rho_\phi$. This system of equations represents two coupled harmonic oscillators with mass terms which, during the

time interval when ϕ is oscillating, contain some periodically varying coefficients. We can diagonalize this system of equations to get two decoupled equations of Mathieu type [7, 15].

$$\frac{d^2Q_k}{dz^2} + [A(k) - 2q\cos 2z]Q_k \;=\; 0\,, \tag{55}$$

where Q stands for one of the eigenvectors of the diagonalization process and $z = mt$.

We need to estimate the values of $A(k)$ and q once ϕ begins to oscillate and can be approximated as $\phi = \phi_e \sin mt$ where ϕ_e is the amplitude of the scalar field at the end of inflation. While for $q \ll 1$, the resonance occurs only in narrow bands [7], for $q > 1$, the resonance occurs for a broad range of values of k [13]. Our numerical simulations indicate that we are in the broad resonance region. Let us for a moment neglect the coupling terms between the Q_f and Q_ϕ equations. In this case, the value of q in the Q_f equation would be of the order

$$q \;\sim\; 6\pi\left(\frac{\phi_e}{m_{pl}}\right)^2 \tag{56}$$

and would be less than 1. However, if we estimate the amplitude $\tilde{q}$ of the periodically varying coefficient in the cross coupling term in Eq. (48), we find

$$\tilde{q} \;\sim\; \frac{H}{m}\frac{\phi_e}{m_{pl}}\,. \tag{57}$$

Inserting the value of H at the beginning of inflation, we obtain a number which is of order unity. Looking at the Q_ϕ equation we find a very similar result: the periodically varying coefficient multiplying Q_f leads to a value of $\tilde{q}$ given by Eq. (57) and hence appears to contribute to the sourcing of the broad resonance. The problem with this argument, however, is that the value of H at the end of inflation has decreased to a value smaller than m, and hence the above estimate of $\tilde{q}$ in Eq. (57) gives a value smaller than 1. Returning, however, to the general equations in the Appendix, we see that there are coefficients which contribute to q which are enhanced by c_s^{-1} and hence, during the period when the equation of state transits from that of inflation to that of regular radiation, leads to an enhancement of q.

To summarize the discussion in the above paragraph: We have presented analytical estimates which support the numerical results which imply that the parametric amplification of the coupled Q_f and Q_ϕ system is in the broad resonance domain, as shown in Fig. 5.

Note that the Q_f and Q_ϕ equations have a singularity when $c_s^2 = 0$. However, we are numerically solving not the Q equations, but the Φ equation, and our evolution equations are non-singular.

The amplification of Q_ϕ and Q_f also gives rise to a parametric amplification of ζ, as can be seen in Fig. 5. From Eq. (45), it can be seen that the large growth of ζ is generated through the isocurvature source term. We have compared the magnitudes of the curvature perturbation and the isocurvature source term in Fig. 6. This confirms numerically that the amplification of ζ occurs once the isocurvature source has become sufficiently large.

As a further consistency check, we can perform a rough estimate of the duration of the period of oscillations until the amplitude of the curvature perturbation can start to grow. As soon as the scalar field starts to oscillate, the isocurvature perturbation in Eq. (45) grows exponentially as $\exp(\mu m(t - t_e))$ where t_e is the time when the oscillation begins and the Floquet exponent μ describes the rate of exponential growth of the instability. For $q \gg 1$, unstable modes grows extremely rapidly with $\mu \sim 0.2$ [13]. Then after integrating Eq. (45), the curvature perturbation becomes

$$\zeta(t) \simeq \zeta(t_e) - \frac{H(t_e)p\Gamma(t_e)}{\mu m(\rho_f(t_e)(1 + w_f(t_e)) + \dot{\phi}(t_e))} e^{\mu m(t-t_e)} \,. \tag{58}$$

The time t when the increase in the curvature perturbation becomes visible compared to its initial value is given by $\Delta\zeta \equiv \zeta(t) - \zeta(t_e) \sim \mathcal{O}(1)\zeta(t_e)$ at the time when the amplitude of the curvature perturbation begins to grow. Then we can estimate the duration of the oscillatory period until the curvature perturbation starts to increase substantially compared to their initial value

$$t - t_e \simeq \frac{1}{\mu m} \ln\left(3\mu m \frac{\zeta(t_e)}{p\Gamma(t_e)} \frac{m^2\phi_e^2}{H(t_e)}\right) \tag{59}$$

where we have used $\rho_f(t_e) \simeq \rho_\phi(t_e), \dot{\phi}(t_e) \sim m\phi(t_e)$, and $w_f(t_e) \sim \mathcal{O}(1)$. $m \sim 10H(t_e)$, $\phi(t_e) \sim 10^{-1}m_{pl}$ and $\zeta(t_e)/p\Gamma(t_e) \sim 10^2$ at the beginning of oscillation for the parameter values in Fig. 6. With these parameters, Eq. (59) gives

$$t - t_e \sim \frac{6}{\mu m} \sim 6\frac{1}{\mu}\frac{H(t_e)}{m}H^{-1}(t_e) \sim 3H^{-1}(t_e)\,. \tag{60}$$

From the numerical calculations (Fig. 6), $(t-t_e)_{numeric} \simeq \Delta \ln(a/a_0)H^{-1}(t_e) \sim 3H^{-1}(t_e)$. This is consistent with the above analytical estimate.

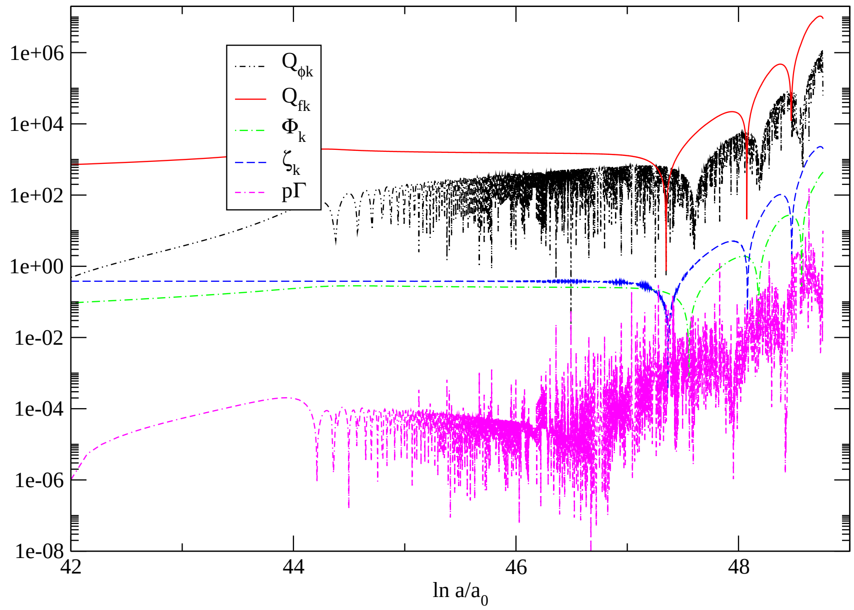

FIG. 7: Evolution of Q_{fk} for the NC fluid, $Q_{\phi k}$ for the scalar field, the Bardeen potential Φ_k, the curvature perturbation ζ_k, and $p\Gamma$ during the inflationary period for the parameter values $\lambda^{-1} = 10^{-4} m_{pl}$ and $\alpha = 1.099$. We set $\phi_i = 10^{-2} m_{pl}$, $\dot{\phi}_i = 0$, and $m^2/H^2 = 10^{-2}$.

We end our discussion by showing the results of a simulation with different values of the parameters. In Fig. 7, we plot $Q_{fk}, Q_{\phi k}, \Phi_k, \zeta_k$ and $p\Gamma$ for $\phi_i = 10^{-2} m_{pl}$ and $m^2/H^2 = 10^{-2}$. Once again we find parametric amplification of the curvature perturbation.

V. DISCUSSION

In this paper we have studied the evolution of cosmological fluctuations through the transition between inflationary phase and radiation phase in our non-commutative inflation model [3, 5]. In this model, there is no traditional reheating phase with oscillating inflaton field. Thus, if there is no matter entropy mode which oscillates, there is no possibility for a resonance growth of curvature fluctuations on super-Hubble scales. As we show here, if a scalar matter field happens to be oscillating at the end of inflation, which is expected to happen in many models beyond the Standard Model of particle physics with light moduli fields (with mass of the order of the Hubble constant during the inflationary phase), then there can be parametric resonance of curvature fluctuations at the end of inflation on super-Hubble scales. This resonant growth happens provided that the scalar field mass at the end of the period of inflation is comparable to the Hubble rate at that time.

The resonance we study here is different from the usual resonance in scalar field-driven models, in which the resonance is driven by the coherent oscillations of the dominant component of matter. In our situation, it is oscillations of a sub-dominant mode which is sourcing the resonance of the curvature fluctuations. We show that there is an entropy mode which undergoes resonance, and once the entropy mode has acquired a sufficiently large amplitude, a resonant growth of the curvature fluctuations is induced.

A characteristic feature of our non-commutative inflation model is the fact that the fluid speed of sound c_s^2 is negative during the phase of accelerated expansion. This appears to lead to an instability of fluctuations on small scales. Since this work focuses on the transition of fluctuations through the period when the equation of state of the background changes, rather than on the question of initial conditions for the inflationary phase, we here to not address this problem. However, we will need to come back to this issue in future work.

Acknowledgments

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APPENDIX A: EQUATIONS FOR Q_f AND Q_ϕ FOR A MODEL CONSISTING OF ONE MATTER FLUID AND ONE SCALAR FIELD

The Sasaki-Mukhanov variables for the fluid and the scalar field are defined in Eqs. (19) and (20), respectively. They satisfy the following set of coupled equations:

$$
\begin{aligned}
&\ddot{Q}_{\phi k} + 3H\dot{Q}_{\phi k} - 4\pi G\frac{\dot{\phi}}{Hc_{sf}^2}(1-c_{sf}^2)\sqrt{\rho_f(1+w_f)}\dot{Q}_{fk} + \Bigg[\frac{k^2}{a^2} + V_{\phi\phi} + 16\pi G\frac{\dot{\phi}}{H}V_\phi \\
&\quad - 16\pi^2G^2\frac{\dot{\phi}^2(1-c_{sf}^2)}{H^2c_{sf}^2}\rho_f(1+w_f) - 32\pi^2G^2\frac{\dot{\phi}^2}{H^2}\rho_f(1+w_f) - 32\pi^2G^2\frac{\dot{\phi}^4}{H^2} \\
&\quad + 24\pi G\dot{\phi}^2\Bigg]Q_{\phi k} - \Bigg[8\pi G\frac{V_\phi}{H} + 6\pi G\dot{\phi}\frac{(1-c_{sf}^2)^2}{c_{sf}^2} + 24\pi G\dot{\phi} \\
&\quad - 16\pi^2G^2\frac{\dot{\phi}(1+c_{sf}^2)}{H^2c_{sf}^2}\rho_f(1+w_f) - 32\pi^2G^2\frac{\dot{\phi}^3}{H^2}\Bigg]\sqrt{\rho_f(1+w_f)}Q_{fk} \;=\; 0,
\end{aligned}
\tag{A1}
$$

$$
\begin{aligned}
&\ddot{Q}_{fk} + \left[3H - \frac{\dot{c}_{sf}^2}{c_{sf}^2}\right]\dot{Q}_{fk} + 4\pi G\frac{\dot{\phi}}{H}(1-c_{sf}^2)\sqrt{\rho_f(1+w_f)}\dot{Q}_{\phi k} + \Bigg[\frac{k^2c_{sf}^2}{a^2} + 6\pi G\rho_f(1+w_f)(1+3c_{sf}^2) \\
&\quad - 6\pi G\dot{\phi}^2(1-c_{sf}^2) + \frac{9}{4}H^2(1-c_{sf}^2)(1+c_{sf}^2) - 32\pi^2G^2\frac{\rho_f^2(1+w_f)^2}{H^2} \\
&\quad - 16\pi^2G^2\frac{\dot{\phi}^2}{H^2}\rho_f(1+w_f)(1+c_{sf}^2) - \frac{3}{2}H\dot{c}_{sf}^2 - \frac{3}{2}H(1-c_{sf}^2)\frac{\dot{c}_{sf}^2}{c_{sf}^2} + 4\pi G\frac{\rho(1+w_f)}{H}\frac{\dot{c}_{sf}^2}{c_{sf}^2}\Bigg]Q_{fk} \\
&\quad - 4\pi G\sqrt{\rho_f(1+w_f)}\Bigg[(1+c_{sf}^2)\frac{V_\phi}{H} - 8\pi G\frac{\dot{\phi}}{H^2}\rho_f(1+w_f) + 3\dot{\phi}(1+c_{sf}^2) - 4\pi G\frac{\dot{\phi}^3}{H^2}(1+c_{sf}^2) \\
&\quad + \frac{\dot{\phi}}{H}\frac{\dot{c}_{sf}^2}{c_{sf}^2}\Bigg]Q_{\phi k} \;=\; 0,
\end{aligned}
\tag{A2}
$$

where we have used $c_{sf}^2 = c_{af}^2$. Similar equations were derived in [27] for constant $w_f = c_{af}^2 = c_{sf}^2$ and with a friction term. Note that Eq.(A2) becomes a single fluid equation for $\phi = 0$. And Eq.(A1) agrees with the equation for a single scalar field equation when $\rho_f = 0$.

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